Site Location for Optimal Triangulation

A Technical Application Note from Doppler Systems

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1.0 Introduction

This application note discusses the siting of radio direction finders for optimal triangulation. This problem was first solved by Gary C. White and published in 1985 in the paper, "Optimal locations of towers for triangulation studies using biotelemetry", in the Journal of Wildlife Management, Vol. 49, pages 190-196. It is also discussed in the book, "Analysis of Wildlife Radio-Tracking Data" by White and Garrott, 1990, ISBN 0-12-746725-4.

It should be noted that many considerations affect the siting of direction finders, including availability of locations, local obstructions to the signal path and received signal strength. In this application note, it is assumed that the terrain is flat, and that the received signal strength is adequate for direction finding anywhere in the target area. It is further assumed that all direction finders are identical and that they provide bearings which are unbiased with an error that may be characterized statistically by a standard deviation of 2 degrees. This corresponds to the Doppler Systems Series 6000 direction finder with fixed site antennas DDF605X.

2.0 Estimation of Maximum Likelihood

The use of contours of probability to express the results of DF triangulation is surprisingly old, and was adopted by the allied forces for long range navigational purposes in World War 2.

Multiple bearings can be combined using the maximum likelihood estimate to determine the location of the transmitter. This estimator was developed by Lenth in 1981 and published in the paper "On finding the source of a signal" in Technometrics, Vol. 23, pages 149-154. It is also described in White and Garrott's book. Because the bearings themselves are random variables, the location of the transmitter is also a random variable which has a two dimensional probability distribution. An ellipse can be calculated which represents the region that has a probability of 95% of enclosing the transmitter. This ellipse will be different at various locations within the target area. For example, at a location remote from all of the direction finders, the bearings will become nearly parallel and the ellipse will become elongated in their direction.

3.0 Area of Ellipse of 95% Confidence

The target area can be normalized to a unit square and the true target location can be selected at some point within this unit square. The area of the 95% confidence ellipse is calculated for this condition. This process is repeated for every other possible location of the target within the target area, and the average of all the 95% confidence ellipses calculated. (In the results presented in this note, we divided the square into a grid of 100 x 100 or 10,000 possible locations and calculated the area of the 95% confidence ellipse at each of them).

Alternative direction finder arrangements (different numbers and locations of sites) can then be compared on the basis of their average 95% confidence ellipse areas. White presents the optimum locations for 2 through 6 sites in his paper. However, in reviewing his results, it is clear that for 3 or more sites, the optimum location is very nearly to place all the direction finders in a circle, spacing them uniformly around the circle. The diameter of this circle increases slightly as the number of direction finders increases, but it is in the range 75% to 80% of the side of the square.

When only 2 direction finders are used, they should be located slightly outside the square along one side of the square. The sites should be located at 10% and 90% of the length of the side in from the corners of the square.

In the following table, the results are summarized for 2 through 10 direction finder sites.

Number of DF Sites, N	Optimal Location (Radius of Circle), Rdf	Average Area of 95% Confidence Ellipse, Aavg	Radius of Circle having Same Area as Average 95% Confidence Ellipse, Ravg
2	X1 = .1, Y1 = 1.01 X2 = .9, Y2 = 1.01	0.017	0.073
3	0.355	0.003031	0.031
4	0.355	0.001861	0.024
5	0.375	0.001359	0.021
6	0.375	0.001043	0.018
7	0.395	0.000845	0.016
8	0.395	0.000711	0.015
9	0.395	0.000605	0.014
10	0.405	0.000530	0.013

The right column lists the normalized linear distance that corresponds to the radius of a circle which has the same area as the average area of the 95% confidence ellipse.

4.0 Application of the Table - Estimating the Location Accuracy

As an example of the use of these results, suppose that 4 direction finders are to be located to cover a region that is 5 miles square. The optimum location for the 4 DF's is to space them uniformly around a circle (centered on the square target area) with a diameter of 5 x 2 x 0.355 = 3.55 miles. The average area of the 95% confidence ellipse will be 5 x 5 x 0.001861 = 0.0465 square miles. The linear error corresponding to a circle of the same area is 5 x 0.024 = 0.12 miles.

5.0 Application of the Table - Selecting the Number of DF Sites Required

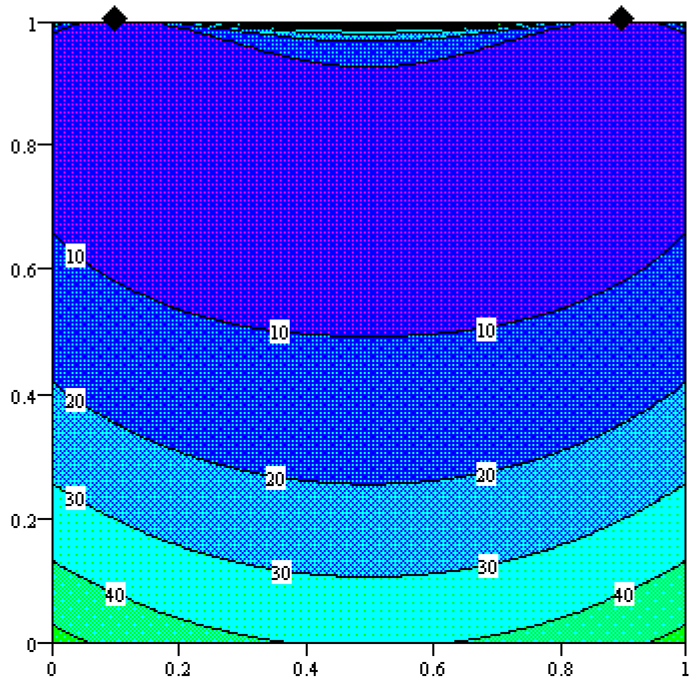
Suppose that the target area is 7 miles square and we require a location accuracy of 0.1 miles. The normalized linear error is 0.1/7 = 0.0143. From the above table, we see that 9 sites are required. These should be spaced uniformly in a circle of radius 7 x 0.395 = 2.765 miles.

6.0 Applicability to Other Direction Finders

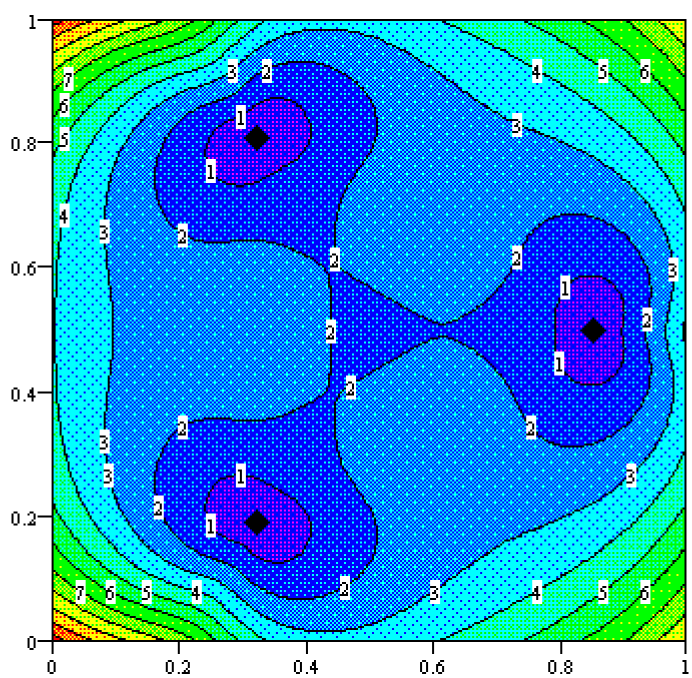
The results presented are based on a direction finder bearing accuracy of 2 degrees (standard deviation) which is typical of an 8 element antenna such as the series DDF605X. To apply these results to a different direction finder antenna, the linear error listed in the table can be ratioed with the standard deviation and the area given in the table can be ratioed with the standard deviation squared - for standard deviations up to 5 degrees. For example, the 5000 series direction finding system uses 4 element antennas and has a standard deviation of 5 degrees. The linear error of a 3 antenna system is then (5/2) x 0.031 = 0.0775. The average area of the 95% confidence ellipse is ((5x5)/(2x2)) x 0.003031 = 0.01894.

7.0 Contour Plots of Area of 95% Confidence Ellipses

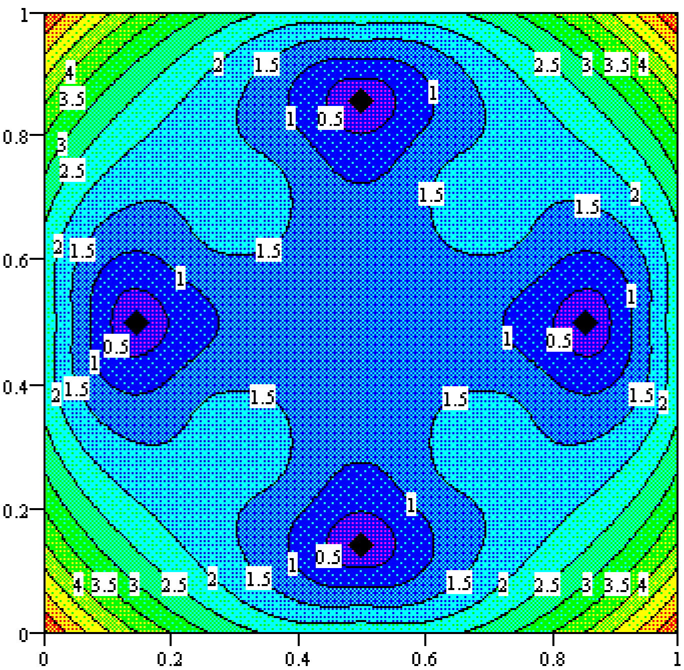
The above table presented the average of the areas of all the 95% confidence ellipses over the entire square target region. It is interesting to see how the actual areas vary over the region. The following figures show this variation in terms of contour of constant area. All of the figures shown in these plots are scaled by 1000. For example, the curve marked "20" in the first plot (2 DF sites) is an "isopleth" which represents the location of all 95% ellipses having areas of 0.020. The location of the two DF sites is indicated by the black diamond symbols.



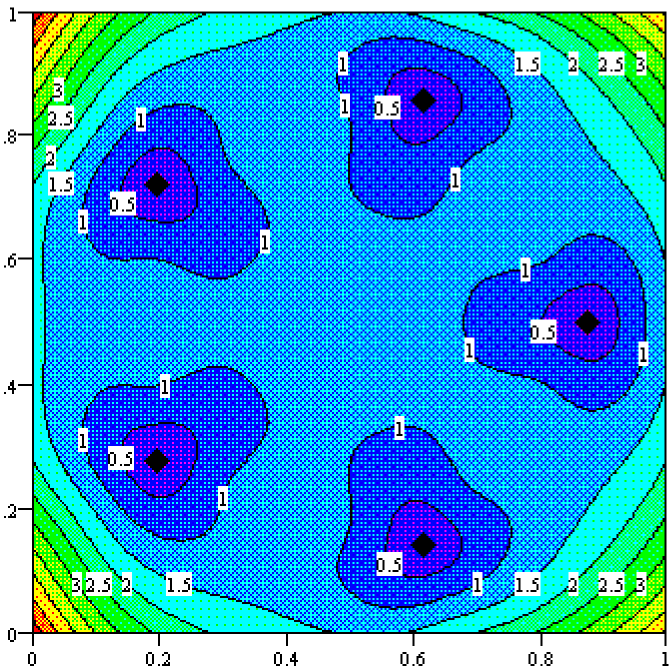
Contour plot of 95% confidence ellipse area for 2 direction finders optimally located on unit square with a standard deviation in the bearings of 2 degrees.



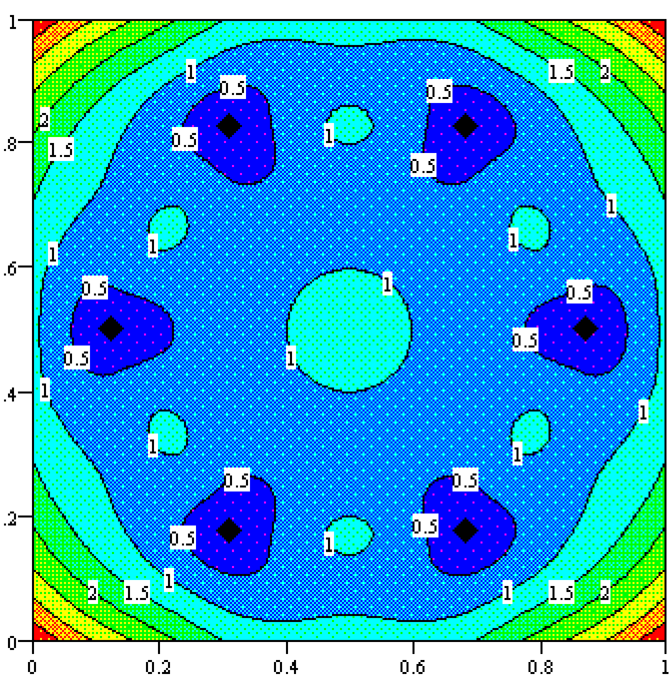
Contour plot of 95% confidence ellipse area for 3 direction finders optimally located on unit square with a standard deviation in the bearings of 2 degrees.



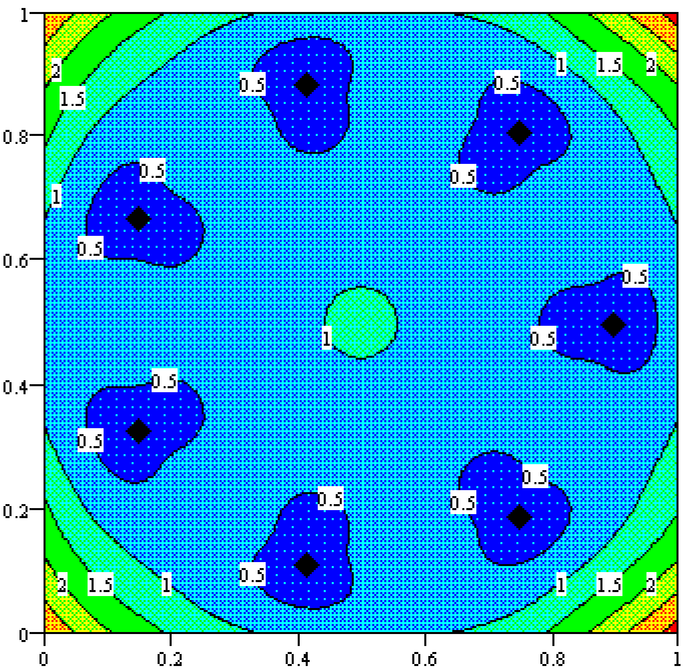
Contour plot of 95% confidence ellipse area for 4 direction finders optimally located on unit square with a standard deviation in the bearings of 2 degrees.



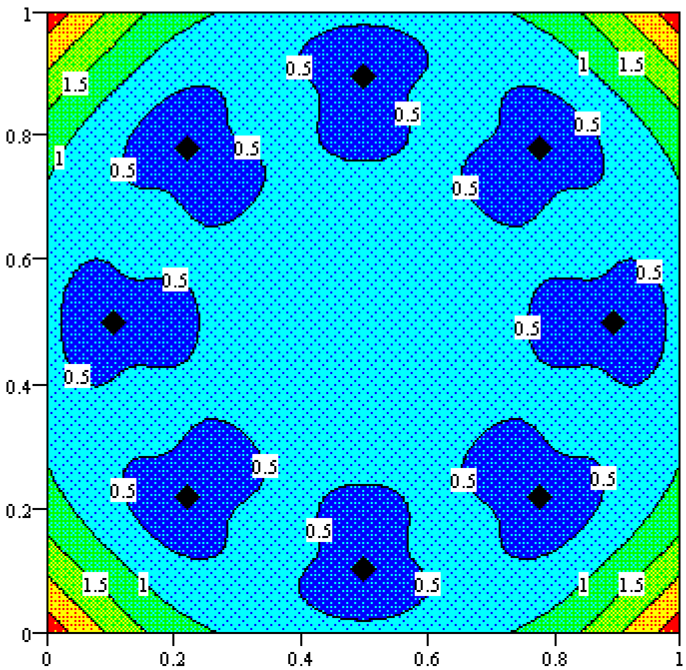
Contour plot of 95% confidence ellipse area for 5 direction finders optimally located on unit square with a standard deviation in the bearings of 2 degrees.



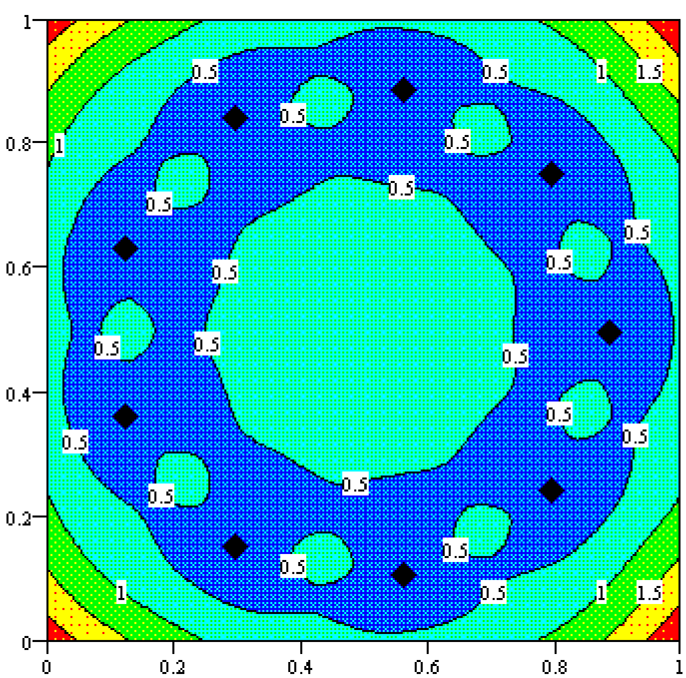
Contour plot of 95% confidence ellipse area for 6 direction finders optimally located on unit square with a standard deviation in the bearings of 2 degrees.



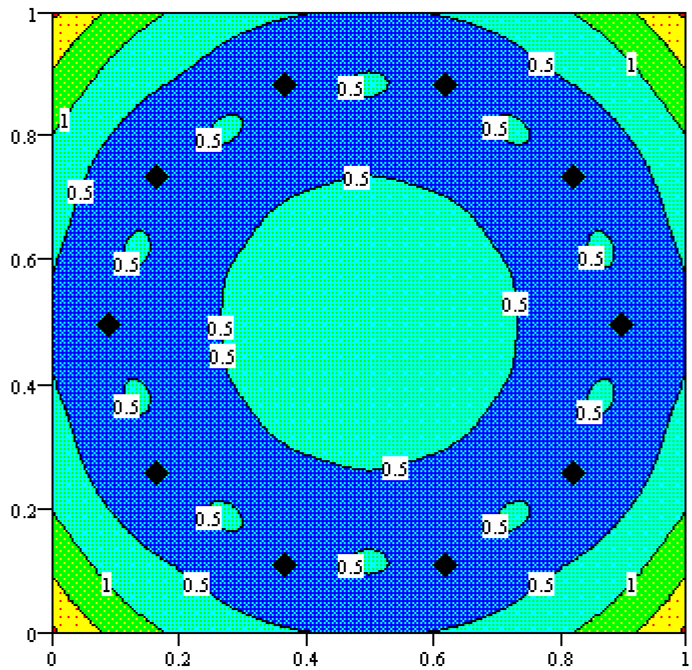
Contour plot of 95% confidence ellipse area for 7 direction finders optimally located on unit square with a standard deviation in the bearings of 2 degrees.



Contour plot of 95% confidence ellipse area for 8 direction finders optimally located on unit square with a standard deviation in the bearings of 2 degrees.



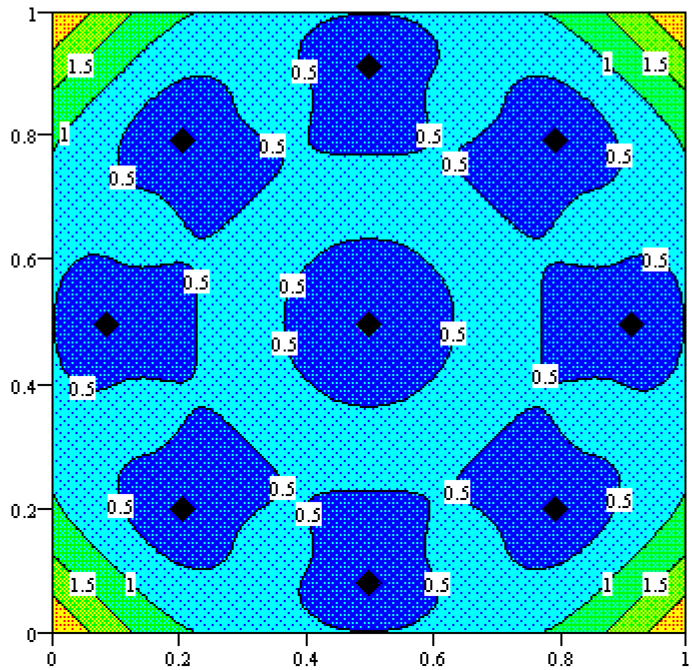
Contour plot of 95% confidence ellipse area for 9 direction finders optimally located on unit square with a standard deviation in the bearings of 2 degrees.



Contour plot of 95% confidence ellipse area for 10 direction finders optimally located on unit square with a standard deviation in the bearings of 2 degrees.

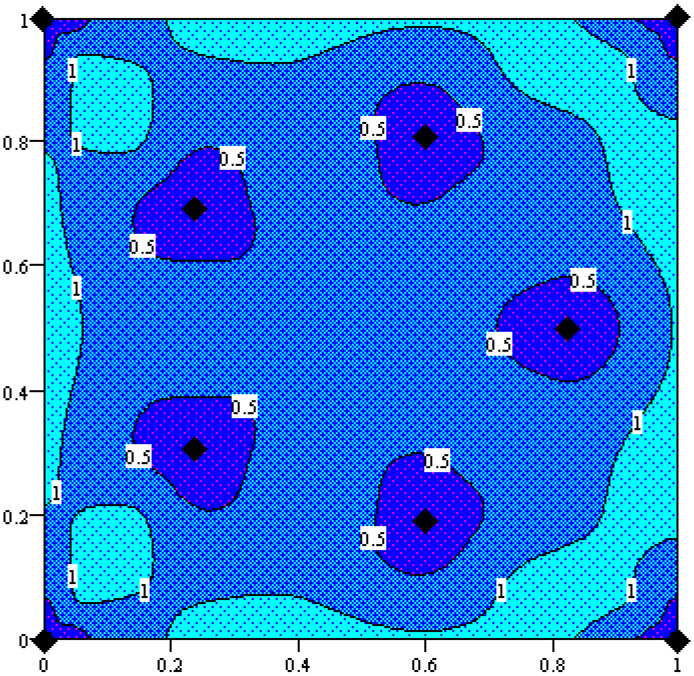
8.0 Alternative Arrangements

In this section, two alternative arrangements are considered. First, we place a DF site at the center of the square and position the remaining sites on a circle. The diameter of the circle is varied until the minimum value for the average area of the 95% confidence ellipses is found. For a total of 9 direction finders, this occurs when the radius is 0.415 and the average area is equal to 0.000606. This is essentially equal to the value of 0.000605 that results when all 9 direction finders are placed on a circle of radius 0.395. When other factors are considered (such as signal strength at the receiving sites), this arrangement may well be preferable. The following figure shows the contour plot.



Contour plot of 95% confidence ellipse area for 1 direction finder at the center and 8 direction finders located in a circle with a standard deviation in the bearings of 2 degrees.

The next case is to place direction finders at the four corners of the square area and position the remaining sites on a circle. Again, taking a total of 9 direction finders, the minimum value for the average area of the 95% confidence ellipses occurs when the circle has a radius of 0.325 and the average area is 0.000793. This is clearly inferior to the above case or to the optimum condition when all 9 sites are on the same circle. The contour plot is shown below.



Contour plot of 95% confidence ellipse area for 4 direction finders at the corners and 5 direction finders located in a circle with a standard deviation in the bearings of 2 degrees.

9.0 Summary

The optimum placement of three or more direction finders in a square area is to locate them uniformly around a single circle having a radius between 75 and 80% of the side of the square. This optimization is based on minimizing the average area of the 95% confidence ellipses which are associated with the estimation of transmitter location using the maximum likelihood method. When signal strength at the received sites is considered, it may be better to locate the sites in multiple concentric circles.