Stress Analysis of Fixed Site Series 6000 Antennas

A Technical Application Note from Doppler Systems

April 12, 2002

1.0 Introduction

This stress analysis considers five different antenna configurations, shown in Figures 1 through 5. Three of these are single antennas, the other two assemblies are a stack of two and a stack of three individual antennas. These are the same five antenna configurations which were used to determine the loading due to a 100 mph wind and ½ inch ice coating in the application note, Wind Loading on Fixed Site Series 6000 Antennas.

Each configuration has similar components in which stresses are of concern. Table 1 summarizes the safety factors for each of these components for all five configurations. The safety factors associated with each component are based on ultimate strength. Since the loading conditions are quite extreme, no other margins of safety are used.

The following sections explain the stress calculations in further detail. In Table 1, "upper" refers to the flange or weld just below the named antenna, and "lower" refers to the bottom of the mast below the named antenna.

The safety factor of "1+" for the whip of DDF6052/6092 is based on an acceptable amount of yield rather than on yield stress. The reason for this is explained in the Whip Stress section.

Table 1 - Antenna Stress Analyses Results

Figure	Case No.	Assembly	Whip	Whip Mount Assy	Arm Root	Hub Ring	Flange	Mast/ Flange Weld	Main Mast Base	Coupling/Bolts
1	1	DDF 6052/6092	1+	7.2	15.3	2.7	None	None	1.15	None
2	2	DDF 6055/6095	11.1	150	130	24	None	None	1.2	None
3	3	DDF 6057/6097	99	1300	1130	200	None	None	1.8	None

4	4	DDF 6052/6092	1+	7.2	15.3	2.7	Upper 1.5	Upper 2.3	1.87	1.3/1.6
4	5	DDF 6055/6095	11.1	150	130	24	Upper 6.7 Lower 1.3	Upper 9.5 Lower 1.3	1.3	3.2/8.9
4	6	DDF 6057/6097	99	1300	1130	200	Lower 6.7	Lower 9.5	10.7	None
5	7	DDF 6052/6092	1+	7.2	15.3	2.7	Upper 2.4	Upper 4.0	1.5	1.8/5.2
5	8	DDF 6055/6095	11.1	150	130	24	Lower 2.3	Lower 2.1	2.3	8.0/2.2

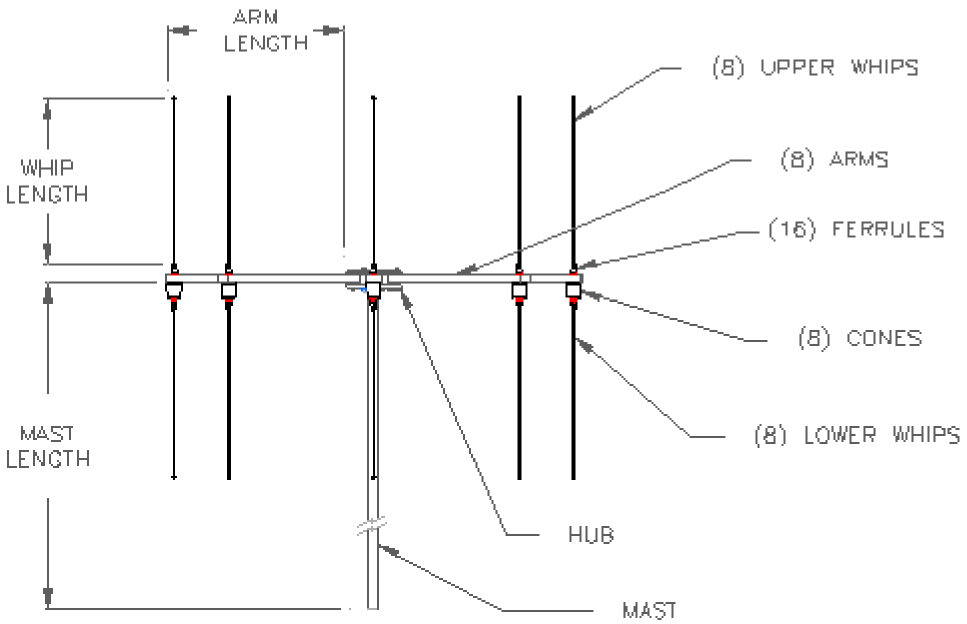


Figure 1 - DDF6052/6092 Antenna

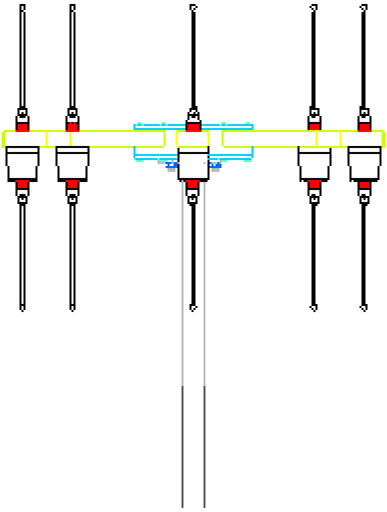


Figure 2 - DDF6055/6095 Antenna

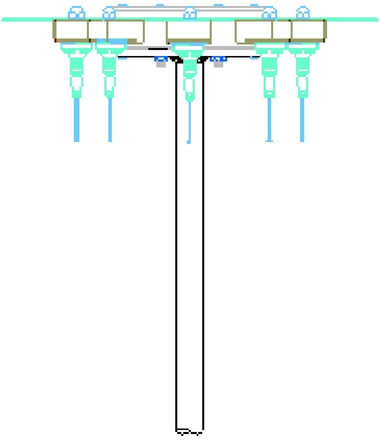


Figure 3 - DDF6057/6097 Antenna

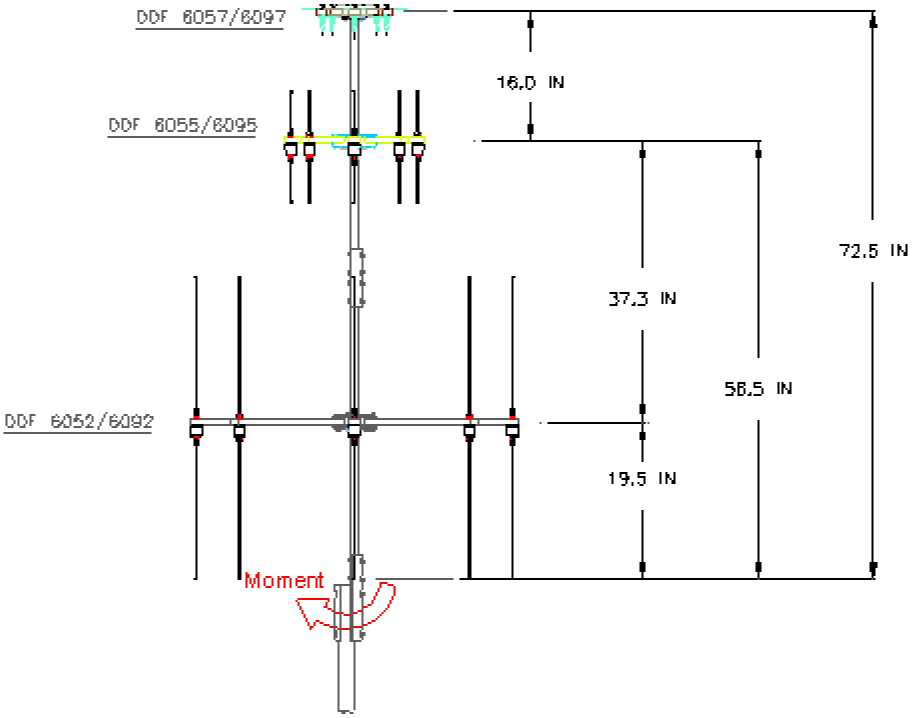


Figure 4 - Three Antenna Stacked Assembly

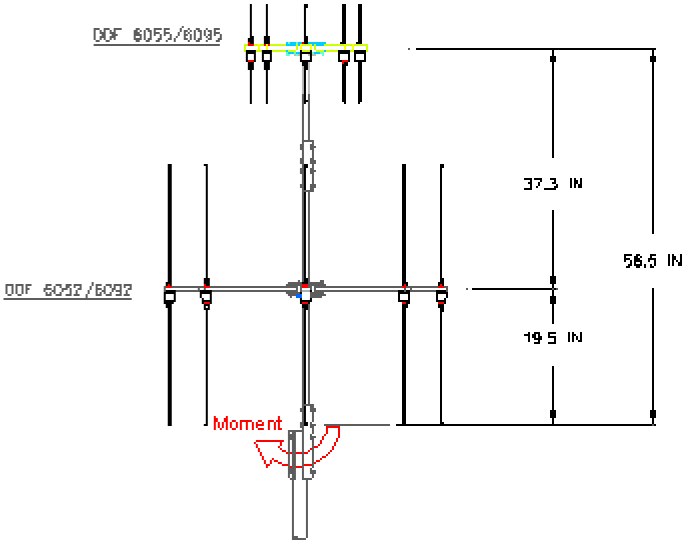


Figure 5 - Two Antenna Stacked Assembly

2.0 Whip Stresses

Various conditions are possible for the whip loading, depending on the wind speed and the distribution of an ice coating. It will be assumed that the wind load will cause the ice to crack away from some length of the whip near the ferrule leaving this part of the whip to support the rest of the whip and its ice. It is also possible that ice would never build up around some length of the whip if wind is present to flex the whip and crack away the ice as it forms. The conditions during ice formation can vary considerably, so in the analysis the length of ice is an independent variable.

Figure 6 illustrates the assumed geometry and gives the nomenclature for the analysis.

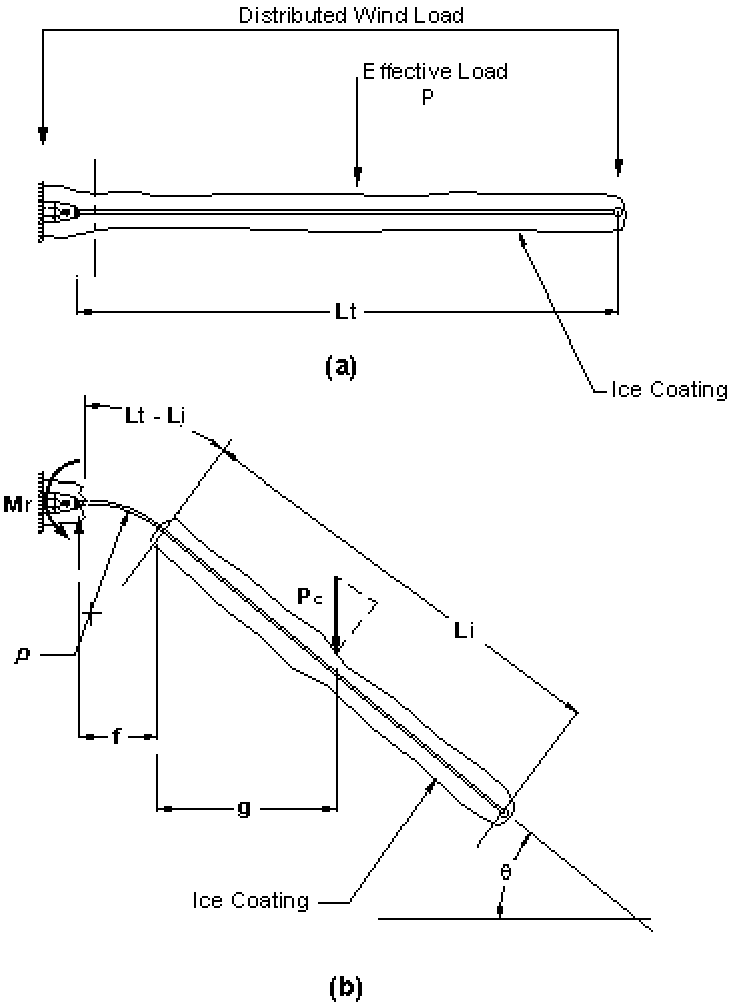


Figure 6 - Whip Configuration and Loads

The following assumptions have been made for the analysis of the whip stresses:

- The wind load is for 100 mph and .5 inch ice thickness (1.1 inch overall whip diameter)
- When the whip is at an angle to the wind, the drag force in the direction of the wind is P_C which is calculated from the equation below, based on the projected frontal area.
- The shear effects are negligible; the moment dominates so the shape of the flexed whip is an arc, Ref 3 .
- Dynamic effects are negligible. Deflections due to flutter or vibrations are small compared to the static deflection.

The procedure used to calculate the deflection and stress starts with the relationship for the moment load, M_a, on the whip as a function of L_i and ρ**.** The restoring moment of the whip, M_r, is a function of ρ. For any assigned length of ice, L_i , these two moments are equated and ρ can be found. The bend angle, θ, can be substituted for the radius of curvature, ρ, as seen below if desired.

The calculations are based on the following values to determine the relationship between length of the ice coating and the stress at the base of the whip.

Total Antenna Length $L_t := 16.5$ in
Whip Shaft Radius $R_w := .05$ in
Youngs Modulus $E := 30 \cdot 10^6$ psi
Wind Load $P := 3.38$ lb

The following equations relate the geometry, the wind load and the stress in the whip.

Iced Length of Antenna $L_i :=$ assigned_variable in

Whip Bend Radius $\rho :=$ indep_variable in

$\theta := \frac{L_t - L_i}{\rho}$ Bend Angle

$P_c := P \cdot \frac{L_i}{L_t} \cdot \cos(\theta)$ Wind Load Component Normal to Unloaded Whip

$M_a := (\mathbf{f} + \mathbf{g}) \cdot P_c$ Applied Moment

$f := \rho \cdot \sin(\theta) \qquad g := L_i \cdot \frac{\cos(\theta)}{2}$ Moment arm lengths

$M_r := E \cdot \frac{I}{\rho}$ Whip Reaction Moment

$$I := \frac{\pi}{4} R_w^{\,4}$$

$\sigma := M_r \cdot \frac{c}{I}$ Resulting Stress also, $\sigma := \frac{E \cdot c}{\rho}$

The above equations can be solved for the stress, σ, and bending angle, θ, as a function of the ice length, Li. Figures 7 and 8 show the results.

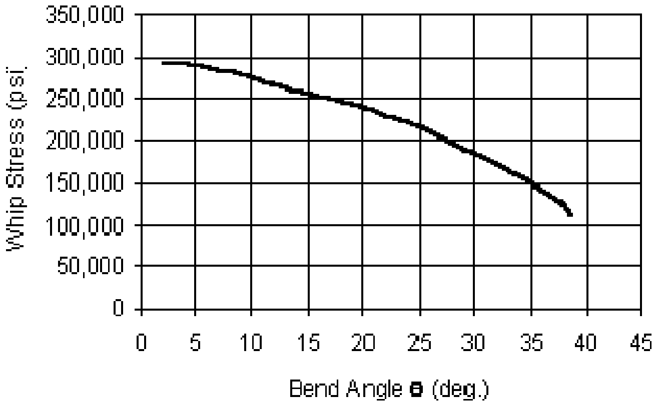


Figure 7 - Whip Stress vs Bend Angle

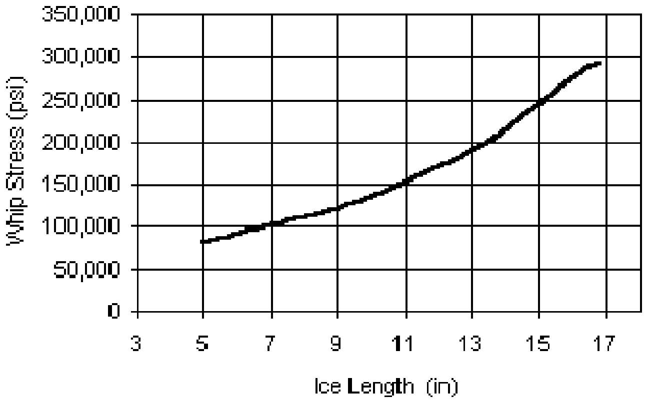


Figure 8 - Whip Stress vs Ice Length

As an example, if the ice length is 15 inches, a stress of about 250,000 psi results and the corresponding bending angle for this stress is about 15°. The worst case is when L_i is near 17 inches (a small length of whip is exposed), where the stress reaches about 290,000 psi and the bend angle is 5°.

The whip material is 18-8 CRES which has a published tensile strength of 81 ksi in the annealed condition, but it is hardenable by cold working, and its strength can reach 250,000 psi or more. It is very likely that this whip material is drawn and cold worked.

Testing was done to learn about its condition. A sample of the material was bent and constrained to a 5.25 inch radius, which corresponds to an apparent stress (using the σ vs. ρ relationship) of nearly 300,000 psi if no yielding occurs. After this test, the residual bend in the material was measured and found to have a radius of curvature of about 500 inches indicating a small amount of yielding and validating the stress assumption. To determine how this relates to an antenna, calculations were made to find the whip tip offset after a whip is subjected to stress at this level. Table 2 shows that for large ice lengths where the highest stress occurs, less than .050 inch permanent deformation occurs.

So even though the safety factor based on an ultimate strength of 250,000 psi (Ref. 4) is less than one, yielding occurs which redistributes the stress and increases the load carrying ability (Ref.3, up to 70% for a circular cross section). Test samples have been bent to radii less than an inch without fracture although the yield was large.

Table 2 - Permanent Whip Tip Deflection After Bending to Yield At 300 ksi

L_T-L_I (in)	L_I (in)	θ (deg)	Tip deflection (in)
0.5	16.5	0.05	0.015

1	16	0.10	0.029
2	15	0.21	0.055
3	14	0.31	0.07

3.0 Whip Mount Stress

The whip and its moment load is supported by the mounting arrangement shown in Figure 9. The upper whip is supported by a stud through the thickest part of the arm. The lower whip and cone are supported by a special mount which enables an insulated conductor to be passed through from the whip. The load bearing component is a brass tab which spans the .75 dia. hole in the arm, retaining the threaded antenna mount.

A moment load on the whip results in a force tending to pull the center of the tab outward through the hole and loading the ends of the tab. Of course, the edges of the hole are also stressed.

This mount was tested to failure, breaking the tab at both ends and deforming the arm material where the tab was seated. The arm material began to deform at a load equivalent to a safety factor of 7.2. The tab fractured at a load equivalent to a safety factor of 10.4.

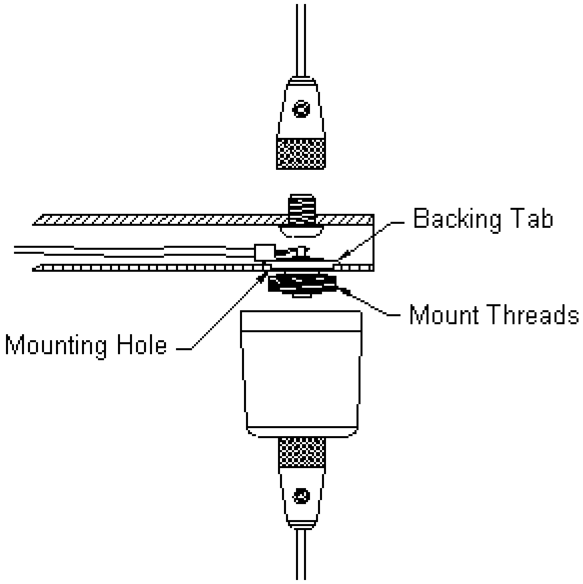


Figure 9 - Whip Mounts on Antenna Arm

4.0 Arm Stresses

The arms support the upper and lower whips and in turn are attached to the central hub. This arrangement is shown in Figure 11 and the arm cross section is shown in Figure 10. Maximum stresses occur at this attachment with the coating of ice and the wind perpendicular to the arm.

All antenna models have arms with the same cross section, but different lengths. Whip lengths vary with arm length. So the stresses on the largest antenna, DDF6052/6092, are the worst case. The arm stresses for the other models can then be scaled using the loads of Ref. 2.

The following equations calculate the arm's section properties and the bending stress where the arm joins the hub.

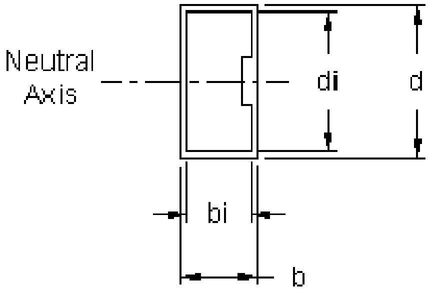


Figure 10 - Arm Cross Section

$d := 1.495$ in

$d_i := 1.371$ in

$b_i := .639$ in

$b := .749$ in

$$I_1 := \frac{b \cdot d^3 - b_i \cdot d_i^3}{12}$$

$I_1 = 0.071$ in^4

$M := 2 \cdot 3.38 \cdot 17 + 2 \cdot .36 \cdot 17 + 1.03 \cdot 17 + 8.69 \cdot 8.5$ in – lb

$M = 218.5$ in – lb

$$c := \frac{d}{2}$$

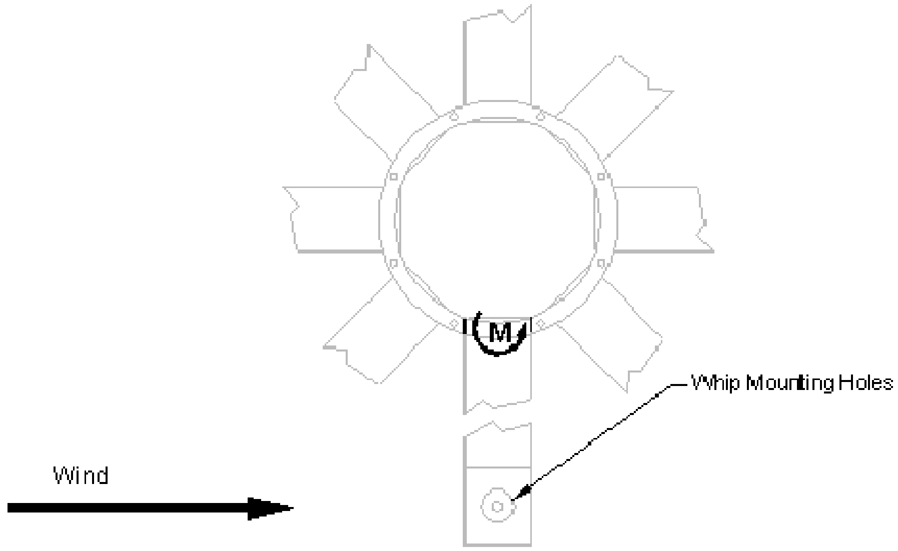
$$\sigma := M \cdot \frac{c}{I_1}$$

$\sigma = 2290.1$ psi

The arms are made of 6061 T6 and are brazed into the hub. The nominal yield strength of this material is 45,000 psi with a slight reduction in strength due to the brazing. Allowing for this, the safety factor is 15.3.

5.0 Hub Stresses

The arm to hub attachments are shown in Figure 11a with details of the hub/arm in Figures 11b through 11d.



(a) Hub/Arm Assembly - Bottom View

(c) Hub Section (b) Hub (d) Hub/Arm Section

Figure 11 - Arm Loading and Attachment

The rectangular cut-out in the hub leaves "beams" of rectangular cross sections A and B. The arm is brazed to these as well as to the short sides of the cutout. An observed accidental failure showed that the braze on the two short sides of the arm may not be significant in sustaining the load and the failures in this case were at the four corners of the cut out. The long sides of the hub cutout remained attached to arm. This leads to modeling the situation simply as a moment M, as shown in Figure 11a, applied to the long sides of the cut out.

Section A

$b := .50 \quad \text{in}$

$h := .375 \quad \text{in}$

$$I_A := \frac{b \cdot h^3}{12}$$

$I_A = 0.002$

Section B

$b := .125 \quad \text{in}$

$h := .375 \quad \text{in}$

$$I_B := \frac{b \cdot h^3}{12}$$

$I_B = 549 \times 10^{-6}$

Total

$I_{total} := I_A + I_B$

$I_{total} = 0.003 \quad \text{in}^4$

$M := 219 \quad \text{in} - \text{lb}$ from the arm stress calculation

$$c := \frac{.375}{2} \quad \text{in}$$

$$\sigma := \frac{M \cdot c}{I_{total}}$$

$\sigma = 14950.4 \quad \text{psi}$

The stress calculated is significantly below the accepted value of 45,000 psi for the T6 condition. A small amount of this strength is lost during the brazing process.

The margins shown together with the fact that the end brazing was neglected gives good confidence in the strength of this joint.

6.0 Flange Stresses

The loading on a specific antenna assembly consists of horizontal wind loads which are grouped into loads on the individual antennas and those on their supporting masts. The moment load on a mast at a specific location is the sum of the moment loads due to the forces above the point of concern. The moment at the top of an antenna assembly is zero and increases toward the bottom due to accumulated loads and increasing mast lengths. Table 3 identifies these points which require analysis.

This section treats the stress in the flanges that join the center hub rings of the antenna to the masts. In the case of an individual antenna on a mast, the only load is a shear load which results in very small shear stresses on the flange, mast and weld but no significant moment. However, in the two and three high stacks the intermediate flanges transfer the moment load downward to the bottom of the lower most mast. Figures 3 through 5 show these flanges and Table 3 below shows their loading and the resulting stresses. "Upper" refers to the flange immediately below the antenna named and "lower" refers to the flange at the bottom of its mast. It should be recalled that the bottom of the lower mast has no flange and that uppermost flanges in each stack have no moment load.

The b/a is the ratio of mast diameter to the flange diameter, β is a parameter dependent on b/a and is used in the equations shown below from Reference 1. M is the applied moment and t is the flange thickness.

$$\sigma_{rMax} := \frac{\beta \cdot M}{a \cdot t^2}$$

Table 3 - Flange Stress Summary

Figure	Case	Antenna	End	Flange Diameter a (inches)	Flange Thickness t (inches)	Mast Diameter b (inches)	Moment M (in-lbs)	b/a	β (Ref.1)	Max σ_r (psi)	Safety Factor
4	4	DDF 6052/6092	Upper	5.5	0.312	1.25	3391	0.227	4.123	26114	1.53
			Lower	None	None	-	-	-	-	-	-
4	5	DDF 6055/6095	Upper	5.5	0.25	1	412	0.182	4.947	5929	6.75
			Lower	5.5	0.312	1	3391	0.182	4.947	31333	1.28
4	6	DDF 6057/6097	Upper	-	-	-	None	-	-	-	-
			Lower	5.5	0.25	1	412	0.182	4.947	5929	6.75

5	7	DDF 6052/6092	Upper	5.5	0.312	1.25	1917	0.227	4.123	16945	2.36
			Lower	None	None	-	-	-	-	-	-
5	8	DDF 6055/6095	Upper	-	-	-	None	-	-	-	-
			Lower	5.5	0.312	1	1917	0.182	4.947	17713	2.26

The safety factors were based on 6061-T6 derated to 40,000 due to welding.

7.0 Mast Stresses

Calculations were made to determine the stress at the bottom of each mast, that is, where it is joined to a customer's mast or where it joins the next lower antenna in the case of an antenna stack.

The following equations were used to determine I and σ. Table 4 shows the results in spreadsheet form. M, the moment load, is taken from Ref. 2.

$$I := \frac{\pi}{4} \cdot \left[\left(\frac{D}{2}\right)^4 - \left(\frac{D_i}{2}\right)^4 \right]$$

$$\sigma := M \cdot \frac{\frac{D}{2}}{I}$$

Table 4 - Stresses at the Bottom of Masts

Case	Location	Tube OD (inch)	Tube ID (inch)	I (in^4)	M (in-lbs)	σ (psi)	Safety Factor
1	Individual 6052	1.25	0	0.120	7,476	39,009	1.15
2	Individual 6055	1.00	0	0.049	3,738	38,094	1.18
3	Individual 6057	1.00	0	0.049	2,451	24,978	1.80
4	3 Stack 6052	1.25	0	0.120	7,687	40,109	1.12
5	3 Stack 6055	1.00	0	0.049	3,391	34,558	1.30
6	3 Stack 6057	1.00	0	0.049	412	4,199	10.72
7	2 Stack 6052	1.25	0	0.120	5,600	29,220	1.54
8	2 Stack 6055	1.00	0	0.049	1,917	19,536	2.30

The safety factors are based on σ_{yield} of 45,000 psi for 6061-T6 aluminum.

8.0 Coupling Stresses

A coupling is used to join the masts of a stacked antenna assembly. It is made up of an aluminum angle bolted through each of the masts to be joined subjecting the length of the angle to bending. Figures 5 and 6 show the coupling locations.

Figure 12 shows the cross section of the load bearing portion of the coupling. Three axes are shown for moment of inertia. The calculations below were done for each and it was determined that axis 1 and distance y_1, gave the smallest inertia value and would therefore be associated with the greatest stress.

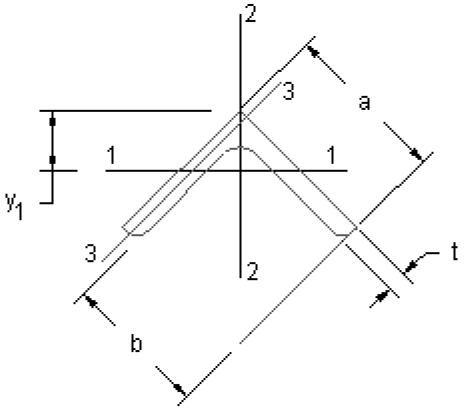


Figure 12 - Coupling Cross Section

$$a := 2\cdot in \qquad t := .25\cdot in \qquad b := (a - t)$$

$$y_1 := 0.7071\cdot\frac{a^2 + a\cdot t - t^2}{2\cdot a - t}$$

$$I_{min} := \frac{a^4 - b^4}{12} - \frac{0.5\cdot t\cdot a^2\cdot b^2}{a + b}$$

Stress results are shown in Table 5 below.

Table 5 - Coupling Stress Summary

Case	Location	Thk. t (in)	Lgth. (in)	b (in)	y_1 (in)	I_{min} (in^4)	Moment Load	Tensile Stress	Safety Factor

							(in-lbs)	(psi)	
4	Three Hi Stack Lower Coupling	0.375	2.00	1.625	0.899	0.206	7800	34,052	1.32
5	Three Hi Stack Interm. Coupling	0.250	2.00	1.750	0.837	0.143	2430	14,175	3.17
7	Two Hi Stack Lower Coupling	0.375	2.00	1.625	0.899	0.206	5600	24,448	1.84
8	Two Hi Stack Interm. Coupling	0.250	2.00	1.750	0.837	0.143	959	5,594	8.04

The two bolts that hold the angle coupling to the masts are loaded in tension. For the case first case shown above the safety factor based on yield for 18-8 CRES bolts is 1.6. Of course, the safety factor will be greater for the other cases since their moments are smaller.

9.0 Weld Stresses

Welds are used to join the masts to the flanges. The weld design is shown in Figure 13.

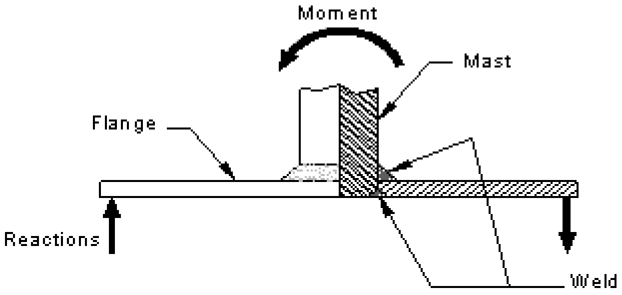


Figure 13 - Weld Design

Prior to welding the mast is inserted into the hole in the flange with a tight clearance fit, so even without the welds the joint can resist moments. A chamfer is provided at the end of the shaft and the ID of the flange as shown to accommodate the bottom weld. Considering the stress in the weld joint, going from the mast to the flange, when the section changes from mast to the mast/weld the cross section area increases, decreasing the stress. Of course, the area greatly increases when the flange itself is reached. So the stresses are bounded by the mast and the flange stresses as treated in previous sections.

10.0 Conclusions

The analysis shows that this family of Doppler antennas is designed to safely withstand winds of 100 mph with a ½ inch coating of ice. An ice free antenna will withstand winds of considerably more than 100 mph.

In general, most of the stressed components are near optimally designed, that is, safety factors are over one but not excessively.

The whips have the lowest safety factor, but as explained in the whip section, this safety factor is based on a tolerable yield and deflection

11.0 References

1. W.C. Young, Roark's Formulas for Stress and Strain, McGraw-Hill, 6th Edition 1989, Table 24, case 20.

2. Doppler Systems Application Note, Wind Loading on Fixed Site Series 6000 Antennas, December 2001.

3. Higdon, Ohlsen, Stiles and Weese, Mechanics of Materials, Wiley and Sons, 2nd Edition, 1967, p. 276 and p. 260.

4. Tensile Elastic Properties of 18-8 Chromium-Nickel Steel as Affected by Plastic Deformation, McAdam and Mebs, NACA Report No. 670, 1938.